Using Probability to Make Decisions

UNDERSTAND In a fair game, each player is equally likely to win. In a fair selection process, each person is equally likely to be chosen. Fairness can be determined by examining probabilities and by comparing the probabilities of winning or being selected for each player. When not playing against others, fairness can be determined by comparing your probability of winning to your probability of losing. For example, flipping a coin is a fair way to decide between two things since the probability of getting heads is equal to the probability of getting tails.

You can also use probabilities to evaluate effectiveness, to decide on problem-solving strategies, or to make an educated decision about a problem. Inventors and companies test their products for effectiveness by gathering and analyzing data in order to make informed decisions. The quality of a test can be evaluated by looking at the relative number of **false positive** and **false negative** results.

UNDERSTAND If a game or process is fair, this does not mean that, in any given number of trials, the results, or **experimental probability**, will directly match the **theoretical probability**. For example, it is possible (though very unlikely) to flip a coin and get heads 10 times in a row. However, if the experiment is repeated many times, there will be roughly the same number of heads as tails. In general, the larger the number of **trials**, the closer the experimental probability will be to the theoretical probability.

A **simulation** is a representative re-creation of a situation or an event. This can sometimes be done by using probabilities. For example, imagine you are in a group of 5 people and one of you will be chosen at random to win a prize. Each person in the group has the same probability of being chosen, $\frac{1}{5}$. You could create a simulation using a spinner that is divided into 5 equal sections, where each section represents a group member. Each time you spin the spinner, the trial will simulate a result. Another way to simulate the situation would be to assign each person a number from 1 to 5 and roll a number cube. If the number cube lands on 6, you would disregard that trial and roll again.

For situations that are not easily simulated using coins, dice, or spinners, you can use random numbers to create simulations. The Random Digits Math Tool on p. 343 has columns of single digits from 0 to 9 arranged in no particular order. You can also use your calculator to generate random numbers. These digits can be used to simulate most situations involving chance behavior. Here are some rules for using random digits for a simulation.

1) Assign numbers to the possible outcomes based on their probabilities. For example, to simulate flipping a coin, assign even numbers to heads and odd numbers to tails.
2) Determine how many digits you will select at a time. For example, to simulate choosing one out of one hundred people, you would need to read every two digits as a number, such as 99 or 05. (The hundredth person could be 00.)
3) Determine which values you will ignore. These are values that are larger than your biggest value or numbers that do not make sense. For example, to simulate rolling a number cube, ignore the digits 7, 8, 9, and 0.

⊏ Connect

Mark and Jorge are playing a game in which they roll two number cubes and add the result. If the sum is odd, Mark gets a point. If the sum is even, Jorge gets a point. Evaluate the game to determine if it is fair. Then, use column 1 of the Random Digits Math Tool on p. 343 to simulate a game with six rolls and see who wins.

1 Make a table to find all of the possible sums.

Count the squares with an odd sum.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2 Determine if the game is fair.

Find the probability of getting an odd or an even sum.

There are 18 odd sums out of 36.

$P(\text{odd}) = \frac{18}{36} = \frac{1}{2}$

There are 18 even sums out of 36.

$P(\text{even}) = \frac{18}{36} = \frac{1}{2}$

$P(\text{odd}) = P(\text{even})$

▸ Since both players have the same probability of winning a point on any given roll, the game is fair.

3 Use random numbers to simulate a game with six rolls.

Each turn consist of rolling two number cubes with numbers from 1 to 6. So, each digit will represent a single roll and the digits 0, 7, 8, and 9 will be ignored. Here are the first 20 numbers from column 1 of the Random Digits Math Tool.

6, 0, 2, 9, 3, 1, 6, 5, 8, 2, 3, 7, 0, 1, 4, 3, 6, 7, 9, 2

First, cross out the excluded numbers.

6, Ø, 2, 9̸, 3, 1, 6, 5, 8̸, 2, 3, 7̸, Ø, 1, 4, 3, 6, 7̸, 9̸, 2

Then, take the remaining numbers in pairs and find their sums. Stop when you have 6 sums.

$6 + 2 = 8$ $3 + 1 = 4$ $6 + 5 = 11$ $2 + 3 = 5$ $1 + 4 = 5$ $3 + 6 = 9$

The random numbers generated 2 even sums and 4 odd sums.

▸ Jorge wins 2 points and Mark wins 4 points, so Mark wins the game in this simulation.

TRY

Simulate another game with six rolls using row 18 of the Random Digits Math Tool. Are the results the same or different? Why?

EXAMPLE A Celiac disease is a disorder of the digestive system that causes discomfort and other symptoms when a person eats certain grains. A doctor is developing a blood test for determining if a patient has celiac disease. The doctor gave the test to 1,000 volunteers, and then confirmed the results via endoscopy. The results are shown in the table below.

Correct Positive	194	Correct Negative	776
False Positive	2	False Negative	28

Evaluate the data and determine whether the new test is reliable. Compare it to a test currently used, which has a false negative rate of about 3.5% and a false positive rate of about 0.5%.

1

Find the probability for a correct result and a false result.

The total number of patients is 1,000.

The total number of correct results is $194 + 776 = 970$.

$P(\text{correct}) = \frac{970}{1000} = 97\%$

The total number of false results is $2 + 28 = 30$.

$P(\text{false}) = \frac{30}{1000} = 3\%$

2

Find the probability for a false positive and a false negative.

The total number of patients is 1,000.

There were 2 false positive results.

$P(\text{false positive}) = \frac{2}{1000} = 0.2\%$

There were 28 false negative results.

$P(\text{false}) = \frac{28}{1000} = 2.8\%$

3

Compare the results to the current test and evaluate the new test.

The probability of a false positive, 0.2%, is less than the current test's false positive rate of 0.5%.

The probability of a false negative, 2.8%, is also less than the current test's false negative rate of 3.5%.

The overall accuracy rate of 97% is very high.

▶ The new test is generally reliable.

TRY

Another blood test for celiac disease was tested on 500 patients already known to have the disease. The results are shown in the table. Evaluate the data and determine whether the test is reliable.

Positive	412
Negative	88

EXAMPLE B A certain professional basketball player has a free throw average of 0.839. Use a calculator to generate random integers from 0 to 999 to simulate free throws. Generate 20 trials and find the experimental probability of making a basket. Then, compare it to the average.

1 Set the parameters of the simulation.

The player's average is 0.839, meaning that he makes free throws an average of 83.9% of the time, or 839 times out of 1,000. We are generating integers from 0 to 999, with a range of 1,000 numbers. The first 839 numbers, that is, numbers from 0 to 838, will be considered making the basket and any number from 839 to 999 will be considered a miss.

2 Prepare your calculator.

Choose a seed (a list of random numbers) by pressing 0 and then [STO▶]. From the home screen, press [MATH] and scroll to the right to access the **PRB** menu. Choose **1:rand** from the menu.

The screen should display 0 ⇒ rand. Press [ENTER].

3 Generate random integers.

Press [MATH] and access the **PRB** menu again. Choose **5:randInt(** from the menu. Enter in the smallest number, 0, followed by a comma and then the largest number, 999, and a closing parenthesis.

The screen should display randint(0,999).

Each time you press [ENTER], a random number will be generated. Let's say we get the following 20 numbers.

943, 900, 146, 514, 405, 733, 43, 339, 995, 200, 798, 951, 220, 369, 7, 935, 108, 6, 548, 855

4 Interpret the results of the simulation and compare probabilities.

Underline all of the numbers in the list that are between 0 and 838.

943, 900, <u>146</u>, <u>514</u>, <u>405</u>, <u>733</u>, <u>43</u>, <u>339</u>, 995, <u>200</u>, <u>798</u>, 951, <u>220</u>, <u>369</u>, <u>7</u>, 935, <u>108</u>, <u>6</u>, <u>548</u>, 855

In all, there were 14 goals and 6 misses out of 20 trials.

$P(\text{goal}) = \frac{16}{20} = 0.8$

▶ The experimental probability of 0.8 is close to but below the actual probability of 0.839. This is because we only did 20 trials in our simulation. If we generated more trials, the experimental results would most likely more closely match the actual average.

DISCUSS

Another player had a career free throw average of 0.8857. Would she beat the player described above in a shoot-off of 10 free throws?

Practice

Fill in the correct word or words to complete each sentence.

1. A game is _____ if each player is equally likely to win.

2. As the number of _____ increases, the experimental probability gets closer to the _____ probability.

3. You can use random numbers to create a _____ for an event involving chance.

Solve.

4. Orit and Amy are playing a game in which they roll two number cubes and multiply the results. If the product is even, Orit gets a point. If the product is odd, Amy gets a point. Is the game fair? If not, who has an advantage? Explain.

   _____

   _____

   _____

5. Harold will take a 10-question true/false quiz. In order to pass the quiz, he must get at least 7 questions right. Harold wants to know if random guessing is a good strategy for taking the quiz. How could you design a simulation to show the possible results of random guessing?

   _____

   _____

   Carry out your simulation. Give the results and determine whether the guessing strategy is effective. Use both the theoretical and experimental probabilities in your explanation.

   _____

   _____

6. There are 10 minutes remaining in a baseball game. The team is losing by 1 run, and the coach is trying to decide whether or not to put in his designated hitter. Out of 12 games this season where they were down by a single run and the hitter was put in, they won 5 and lost 7 games. Out of 8 games this season where they were down by a single run and the hitter was not put in, they won 3 and lost 5 games. Should the coach put in the designated hitter? Use probabilities to explain your answer.

   _____

   _____

   _____

Solve.

7. A coin was flipped 10 times, then 100 times, then 1,000 times. The results are displayed in the table.

Total Flips	Heads	Tails
10	7	3
100	41	59
1,000	512	488

Is the coin most likely a fair coin or a weighted coin? Explain.

_____

_____

8. **SIMULATE** A certain baseball player had a batting average of 0.321. Begin with row 15 of the Random Digits Math Tool to simulate 10 at bats. Calculate the experimental probability of her getting a hit and compare it to the real batting average. Show your work.

_____

9. **ANALYZE** Toxoplasmosis is an infection by a parasite in cats. A veterinarian is developing a test for the parasite. He administers the test to 100 cats whose infection status is known. The results of the tests are listed in the table below. Evaluate the test.

Correct Positive	63	Correct Negative	20
False Positive	8	False Negative	9

_____

_____

_____